Examining the Garren-Kirk Dipole Cooling Ring with Realistic Fields

Steve Kahn

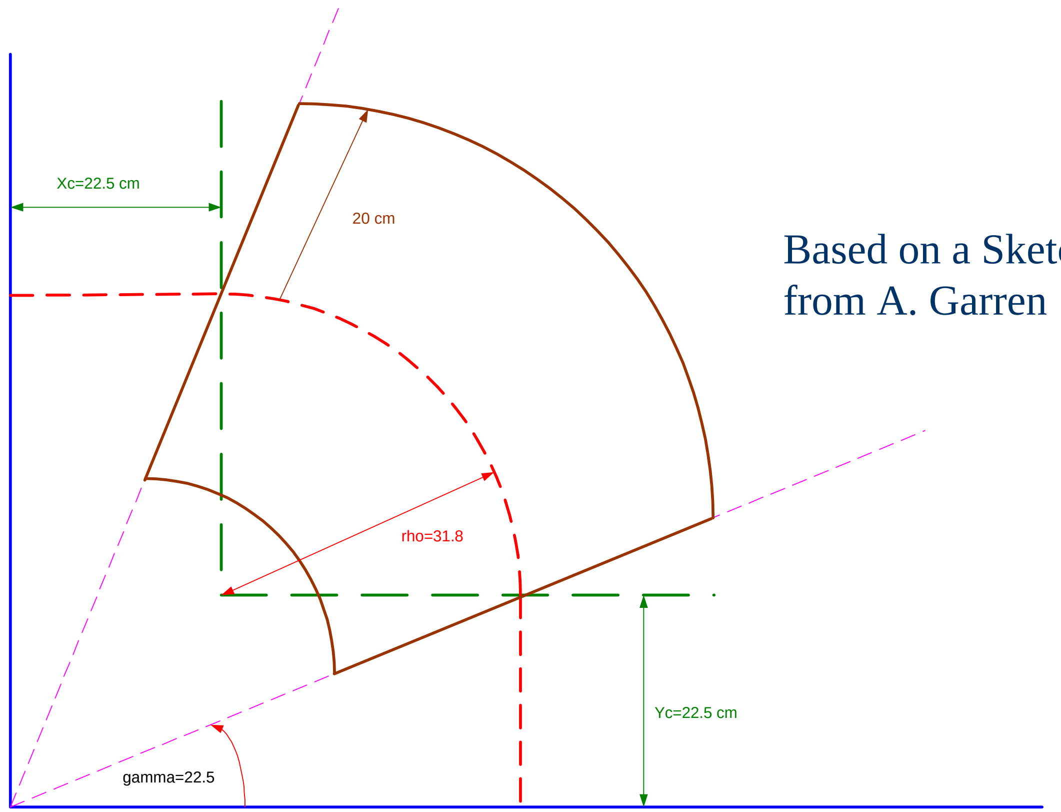
Alper Garren

Harold Kirk

Dipole Ring Parameters

Parameter	Value
Reference Momentum	250 MeV/c
Number of Half-Cells	4
Bend Angle per Half-Cell	90°
Ring Circumference	3.8 m
Number of RF cavities	4
RF Gradient	40 MV/m
Absorber	Pressurized H_2
Hardedge Dipole Field	2.6 T
Straight Length per Half-Cell	40 cm
Dipole Radius of Curvature	31.8 cm

Half Cell Geometry Description



Based on a Sketch from A. Garren

Using TOSCA

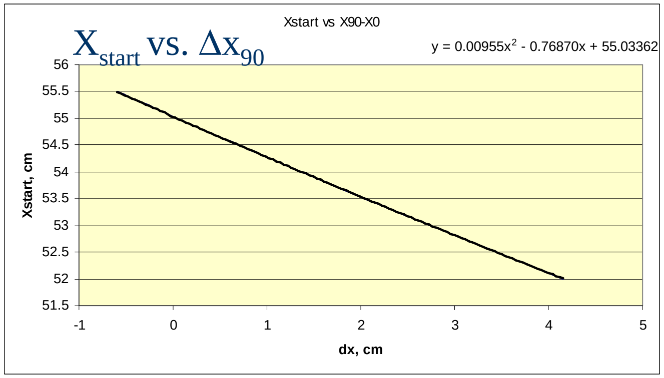
- Hard edge field calculations for the Garren-Kirk Dipole Ring have shown promising results.
  - It is essential to examine the ring using realistic fields that at least obey Maxwell’s equations.
- Tosca can supply fields from a coil and iron configuration.
  - We can use the program to supply a field map that can be used by ICOOL and GEANT.
- Tosca itself can also track particles through the magnetic field that it generates.
  - This allows us to avoid the discretization error that comes from field maps.

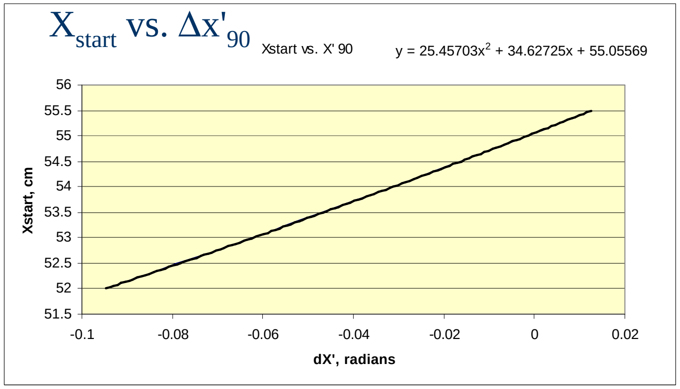
Tosca Model

- For the ease of calculation we are modeling the dipole magnets by its coils only. This may not be the way we would actually engineer the magnet if we actually built it.
  - The field can then be calculated from a Biot-Savart integration directly. No finite-element mesh is necessary if iron is not used.
- There are limitations in the Tosca tracking.
  - Tosca permits only 5000 steps. This limits the step size to ~0.5 mm. This may limit the ultimate precision.

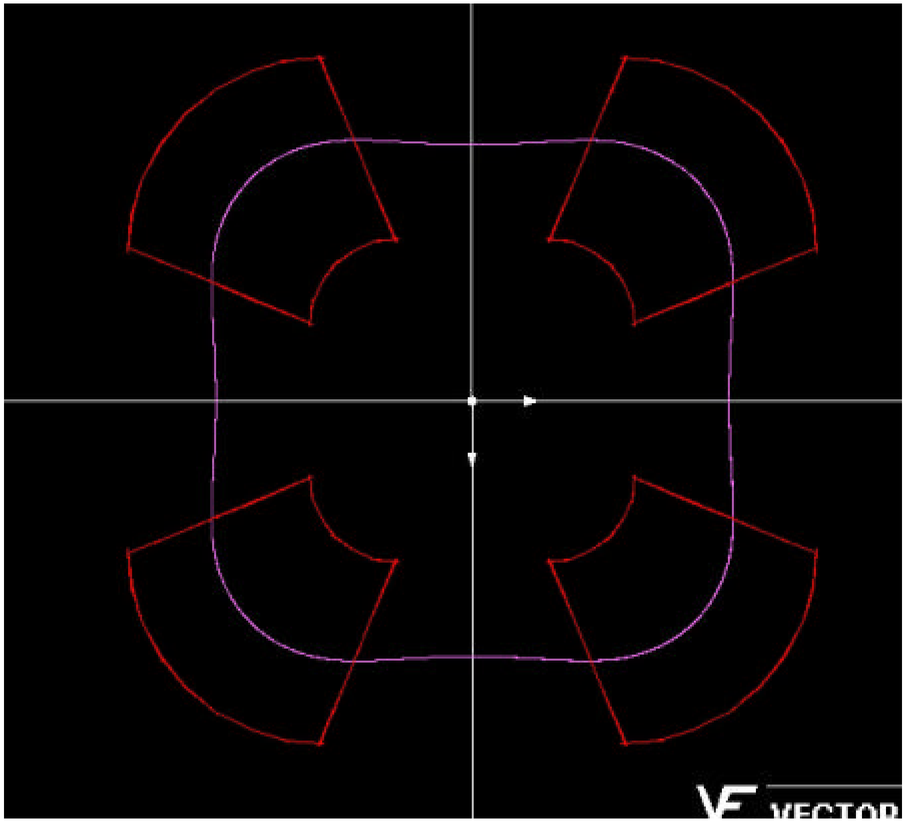
Finding the Closed Orbit

- We know that the *closed orbit* path must be in the *xz plane* and that it must have *x'=0* at the *x-axis* from symmetry.
  - We can launch test particles with different X_{start}.
  - The figures on the right show X_{start} vs. Δx_{90} and X_{start} vs $\Delta x'_{90}$.
    - Where Δx_{90} and $\Delta x'_{90}$ are the variable differences after 90° advance.
    - We find that the best starting values are
      - X_{start}=55.03362 cm for Δx_{90}
      - X_{start}=55.05569 cm for $\Delta x'_{90}$





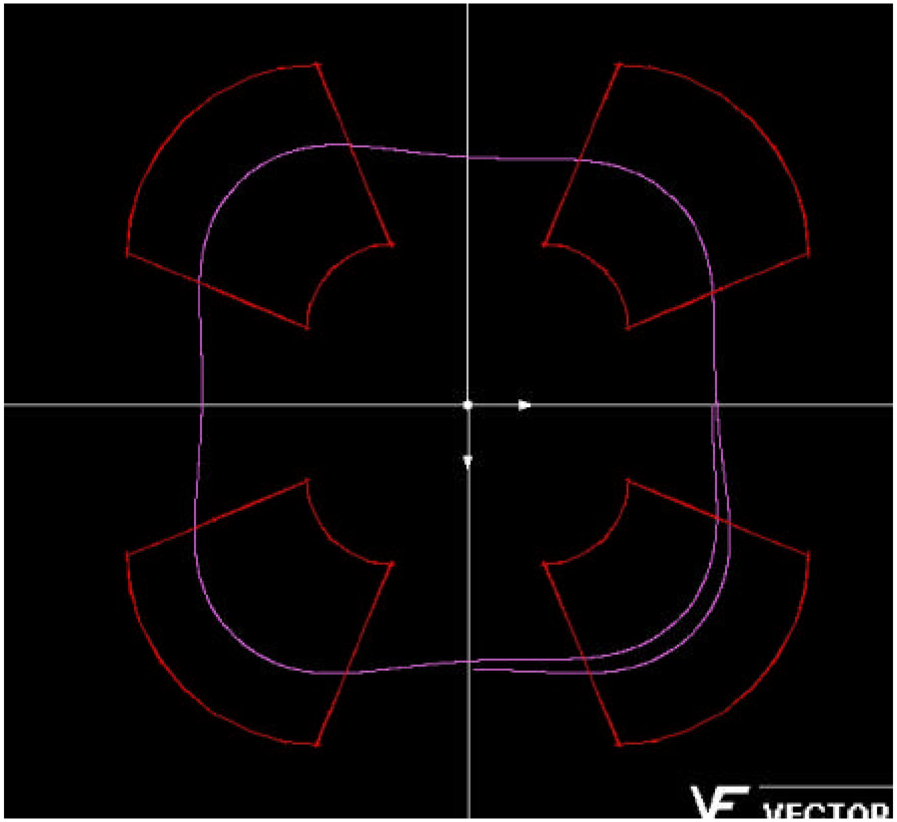
Closed Orbit



Closed orbit trajectory for 250 MeV/c μ started at x=55.02994 cm.

Note that there is curvature in region between magnets since there is still a significant field.

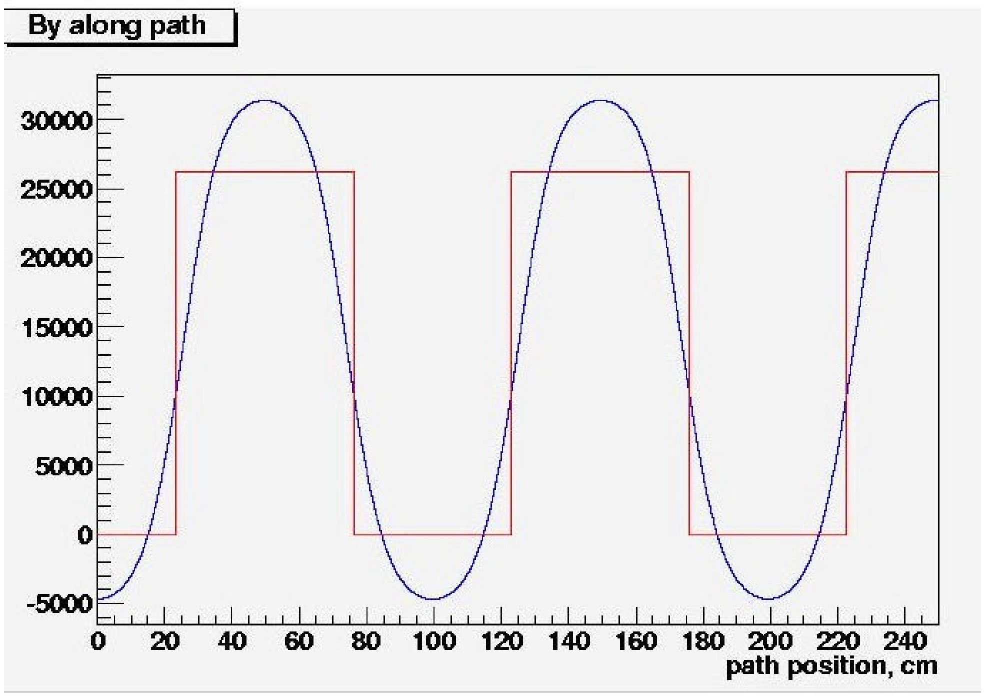
Non-Closed Orbit



Started muon at -2 cm from closed orbit. All other parameters the same.

Field Along the Reference Path

- Figure shows B_y along the 250 MeV/c reference path.
  - The blue curve indicates the field from the Tosca field map.
  - The red curve is the hard edge field.
- Note the –0.5 T field in the gap mid-way between the magnets.



Calculating Transfer Matrices

- By launching particles on trajectories at small variations from the closed orbit in each of the transverse directions and observing the phase variables after a period we can obtain the associated *transfer matrix*.
  - Particles were launched with
    - $\delta x = \pm 1$ mm
    - $\delta x' = \pm 10$ mr
    - $\delta y = \pm 1$ mm
    - $\delta y' = \pm 10$ mr

90° Transfer Matrix

- This is the transfer matrix for transversing a quarter turn:

$$\begin{bmatrix} dx \\ dx' \\ dy \\ dy' \end{bmatrix} = \begin{bmatrix} -0.29145 & 31.965 & 0 & 0 \\ -0.0287 & -0.289 & 0 & 0 \\ 0 & 0 & -0.18336 & 52.9949 \\ 0 & 0 & -0.01823 & -0.1853 \end{bmatrix} \begin{bmatrix} dx_0 \\ dx'_0 \\ dy_0 \\ dy'_0 \end{bmatrix}$$

- This should be compared to the 2×2 matrix to obtain the twiss variables:

$$\begin{bmatrix} \cos m + a \sin m & b \sin m \\ g \sin m & \cos m - a \sin m \end{bmatrix}$$

Twiss Variables Half Way Between Magnets

Parameter	Tosca	A. Garren Synch
m_x	**98.38°**	**99.8784°**
b_x	**32.3099 cm**	**37.854 cm**
a_x	**-0.00124**	**0**
m_y	**100.62°**	**92.628°**
b_y	**53.9188 cm**	**56.891 cm**
a_y	**0.0009894**	**0**